Synopsis for PhD programme

Title of the research:

Some studies on quantum mechanical problems in the framework of wavelet numerical analysis

Name of the candidate: **Manoj Kumar Saha**

Name of the Guide: **Dr. Madan Mohan Panja**

Registration No.: **VB-237 of 1995-96**

Date of Registration: **03-12-2013**

Keywords:*Wavelets, Multiresolution Analysis, Approximate eigen-states*

Wavelets are now being adopted for a vast number of applications, often replacing the conventional Fourier Transform. Many areas of physics have seen this paradigm shift, including ab-initio calculations in molecular dynamics, astrophysics, density-matrix localization, seismology, optics, turbulence and quantum mechanics. This change has also occurred in image processing, ECG analyses, brain rhythms, DNA analysis, protein analysis, climatology, human sexual response analysis, general signal processing, speech recognition, acoustics, vibration,signals, computer graphics, multi-fractal analysis and so on.

Wavelets are functions that satisfy certain requirements. The very name wavelet comes from the requirement that they should integrate to zero oscillating above and below the x axis within a short span. There are various types of wavelets. One can choose between smooth wavelets, compactly supported wavelets etc. We will concentrate only compactly supported wavelet of Daubechies family due to its useful properties which can be employed for the development of efficient mathematical tools for the study of quantum mechanical problems. The Daubechies compactly supported wavelets are introduced by Ingrid Daubechies in 1988 [1,2] using a finite set of non-zero scaling coefficients with $\sum_{l=0\ or-K+1}^{2K-1,K} h_l = \sqrt{2}$, where K denote the order of the Daubechies wavelet.

Here we present some properties of Daubechies wavelets. The orthonormal basis of compact support wavelets of $L^2(R)$ is formed by the dilation and translation of the functions $\varphi_{j,k}(x) = 2^{j/2}\varphi\left(2^j x - l\right)$ and $\psi_{j,k}(x) = 2^{j/2}\psi\left(2^j x - l\right) where j, k \in \mathbb{Z}$.

The function ψ(x) is called a mother wavelet, and $\varphi(x)$ the scaling function.They both satisfy the following two scale relation (refinement equation):

$\varphi(x) = \sqrt{2}\sum_{l=-K+1}^{K} h_l\, \varphi(2x - l)$,

$\psi(x) = \sqrt{2}\ \sum_{l=-K+1}^{K} g_l\, \varphi(2x - l) where g_k = (-1)^k h_{1-k}$.

The set of coefficient $\{\ h_l,\ \ l = 0, \ldots \ldots 2K - 1\ or\ -K + 1 \ldots \ldots K\}$ in the expansion of φ is called low-pass filter and the set of coefficient$\{\ g_l,\ \ l = 0, \ldots \ldots 2K - 1\ or\ -K + 1, \ldots \ldots, K\}$ involved in the expansion of ψ known as high pass-filter.

The supports of the scaling function$\varphi(x)$ and its corresponding wavelets$\psi(x)$ are: for given K, Supp$\varphi = [-K + 1, K]$ and Supp$\psi = [-K + 1\ , K]$.

The filter coefficients satisfy the following condition

$$\sum_{l=-K+1}^{K} h_l = \sqrt{2} and \sum_{l=-K+1}^{K} h_l\, h_{l-2m} = \delta_{0,m}$$

where $\delta_{0,m}$is the Kronecker delta.

The scaling function$\varphi(x)$ and the mother wavelet$\psi(x)$ have the following properties:

Pseudo probability density:$\int_{-\infty}^{\infty} \varphi(x)dx = 1$

Orthonormality ofφ_k:$\int_{-\infty}^{\infty} \varphi_l(x)\varphi_k\,(x)dx = \int_{-\infty}^{\infty} \varphi(x-l)\varphi(x-k)dx = \delta_{l,k}$

Vanishing moment condition; $\int_{-\infty}^{\infty} x^k\, \psi(x)dx = 0 \;\; k = 0, 1, \ldots . K-1$

Orthonormality of $\varphi and\ \psi$: $\int_{-\infty}^{\infty} \varphi(x)\psi(x-m)dx = 0 \qquad m \in Z$

We defineV_J as the closure of the space generated by $(\varphi_{j,k}(x)\ k \in Z\ ) \ and\ W_{j,}$ orthogonal complement of $V_J$in V_{J+1} ,as the closure of the space generated by ($\psi_{j,k}(x)\ k \in Z$).Then

$V_{J+1} = V_J \oplus W_J$.

Here $\oplus$ denotes the orthogonal direct sum.The sub-space $V_J$satisfy the condition: $V_j \subset V_{j+1}$.

Since most of the non-relativistic and relativistic quantum mechanical potential problems describing realistic processes at micro-level are not exactly solvable, we have decided to develop an approximation scheme based on recently developed wavelet numerical analysis to get approximate eigen-states of Schrodinger/Dirac equations involving some potentials of missing eigen-states [3,4] . The term "*missing state*" means, in the analytical solution to some quantum mechanical problems, some eigen-states having number of nodes in between the number of nodes of two consecutive states of available analytical solution have not been observed. It is our goal to explore whether proposed approximation scheme based on wavelet numerical analysis can expose those missing states. We have divided our study into the following steps:

Step-I: Multiresolution analysis of quantum mechanical state space,

Step-II: Multiresolution approximation of functions (potential),

Step-III: Nonstandard representation of differential/quantum mechanical operators,

Step-IV: Reduction of eigenvalue problem involving differential operator (Schrodinger

/Dirac equation) to matrix eigenvalues problem,

Step -V: Finding eigenvalues and eigenvectors of the matrixeigenvalues problem and their inversion to eigenstate of operator eigenvalues problem

Step –VI: Interpretation and conclusion.

References:

[1] I. Daubechies, Commun, Pure Appl. Math. **41** 909-996(1988); Ten Lectures on Wavelets, SIAM. Philadelphia (1992).

[2] A. Cohen, Numerical Analysis of Wavelet Methods, Elsevier, Amsterdam, (2003).

[3] R. Adhikari, R. Dutt, Y.P.Varshni, Exact Solutions of polynomial potentials using supersymmetry inspired factorization method, Phys. Letts. A **141**, 1-8(1989).

[4] David Agboola, Jon Links, Ian Marquette, Yao-Zhong Zhang, New quasi-exactly solvable class of generalized isotonic oscillator, J. Phy.A: Math. Theor, **47** (2014) 395305.